**APEC Math Review - Day 2 exercises**

1. Let a collection of commodities vectors be given as:

You can view commodities as food, clothing, entertainment etc. Each will have multiple elements, for instance food may contain dairy, candy, fresh fruits, and so on. We can call an element of this set as a “bundle.” One property of the consumption set is that it is convex.

Let price be and wealth be . Define this person’s budget set as: . Show that this budget set is convex. (*Hint: use the fact that you know that the consumption set is convex to begin with*).

**Answer:**

This is a direct proof.

Let and be two elements in the budget set, so that and . Denote their convex combination as . Since the consumption set is convex, then evidently . We need to show that , i.e.

By the definitions of and we have and (just multiplying with the relevant scalars)

Putting these two together:

Since , this budget set is convex.

1. Prove that when is a convergent sequence of numbers in , its limit is unique.

**Answer:**

This is an indirect proof.

Suppose that the limit is not unique, so that there are two limits, and . Then for all there exists some such that whenever , and similarly for . Let .

Then by the triangle inequality, we have . But if then we have , which is a contradiction. Q.E.D.

1. Bonus: prove that when is a convergent sequence of elements in a metric space, its limit is unique.

**Answer (almost identical to the previous one, which shows that this is really a property of metric spaces rather than real numbers specifically).**

Suppose that the limit is not unique, so that there are two limits, and . Then for all there exists some such that whenever , and similarly for . Let .

Then by the triangle inequality, we have . But if then we have , which is a contradiction. Q.E.D.

1. Walras’ Law is a fundamental result in consumer theory. Walras’ Law says that under standard conditions on preferences, consumers will choose a consumption bundle on their budget constraint, i.e., they will spend all their money.

Formally: Consider a consumer with locally nonsatiated preferences, and let x\* be their optimal consumption bundle, p a vector of fixed prices, and W their wealth. Then .

This requires a definition of locally nonsatiated, which is:

A consumer’s preferences over consumption set X are considered locally nonsatiated if , every open ball around x contains a bundle that is strictly preferred to x.

Prove Walras’ Law.

**Answer:**

This is an indirect proof.

Suppose that is an optimal consumption bundle but that so that consumer does not spend their entire income. Then we can construct an open ball of radius around which is entirely affordable, so that for each . But by local nonsatiation, each such ball contains a bundle that is strictly preferred to . But then is not the best affordable bundle, which is a contradiction.